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Dispersion in oceanic crust during earthquake preparation

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Abstract

Dispersion of Stoneley waves is studied in a sedimentary layer of ocean bottom resting over basaltic solid half space. Sedimentary layer is assumed a transversely isotropic poroelastic medium. Lower-most solid half-space is assumed to be embedded with vertically aligned saturated micro-cracks and behaves transversely isotropic to wave propagation.

Frequency equation is obtained in the form of determinantal equation. Role of phase angle is eliminated by expressing slowness of waves in terms of phase velocity and elastic constants. Numerical solutions for phase velocity and group velocity are obtained for a particular model. Calculations are made for different depths of ocean and sediments. Effect of thickness and density of cracks on these velocities are observed.

Special cases are discussed which represent the absence of ocean and sediments, in the model considered. Changes in dispersion are discussed during the stress accumulation in an earthquake preparation region.

Keywords: Dispersion; Surface waves; Oceanic crust; Crack density; Transversely isotropic; Poroelastic

1. Introduction

Most of the epicentres of the earthquakes are in oceanic crust. Ocean ridge system is an area of frequent seismic activity. Except near the crest of the mid-ocean ridges, the ocean floor is mantled with sediments. Sedimentary rocks contain water-filled pore spaces after deposition and can be modelled as water saturated porous solid. Sediments in water are believed to be deposited in preferred orientation and behaves transversely isotropic to wave propagation. These rocks are resting on the Igneous rocks which are pervaded by distribution of fluid-filled cracks. Cracks in a region of earthquake preparation modify with the accumulation of stress there. These modifications in the configuration of cracks in a focal region are believed to be the driving mechanism for the precursors of an earthquake.

The theory of effects of cracks on the elastic solids started with the classic paper by Eshelby (1957). The wave velocities for the elastic solids containing cracks have been approximated by Garvin and Knopoff (1973, 1975a, b). O'Connell and Budiansky (1974) and Budiansky and O'Connell (1976) calculated the effects of introduction of cracks on the elastic properties of an

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isotropic solid using self-consistent procedure. Hudson (1980, 1981) developed these ideas further for dilute concentration of cracks, treating cracked solid as anisotropic one. Crampin and co-workers (1978, 1980, 1984, 1987) studied various aspects of wave propagation in cracked solids with presence of aligned crack leading to anisotropy. Crampin (1985) and Crampin and Atkinson (1985) suggested the widespread distribution of aligned cracks in the top 10–20 km of crust. Crampin (1987) explained the effects of stress accumulation before an earthquake, on the modifications of the cracks present.

Seismic anisotropy due to aligned cracks has also been interpreted from the field data in the number of studies (Crampin and Booth, 1985; Crampin et al., 1986; Chen et al., 1987; Lynn and Thomsen, 1990; Lynn, 1991). In recent years, laboratory confirmation of the theory is performed by Rathore et al. (1995). Using these laboratory findings Thomsen (1995) studied the elastic anisotropy due to aligned cracks in porous rocks.

Theory of surface wave propagation in earth models is well understood. After Stoneley (1926) and Tolstoy (1954), a large number of studies discussed the propagation of surface waves in oceanic crust. In some studies (viz. Abubaker and Hudson, 1961), the oceanic crust is considered to be an anisotropic solid. Sharma et al. (1991) studied surface waves in a two-layered model of oceanic crust consisting of poroelastic sediments resting over an anisotropic bed. The recent interpretation of field data confirmed that anisotropy present in the crust may be due to the presence of vertically aligned and saturated cracks. Most commonly occurring anisotropy due to the presence of cracks is transverse isotropy. Hence, poroelastic sedimentary rocks overlying a fluid saturated cracked elastic solid half-space may be a realistic model for oceanic crust which behaves transversely isotropic to wave propagation. Assuming the changes in dispersion behaviour of surface waves as a possible precursor for an impending earthquake, in the present study, I propose to study the effects of modifications of cracks in the oceanic crust on the dispersion of Stoneley waves.

2. Geometry of the medium

A three-layered medium is considered with the uppermost layer of homogeneous liquid (medium I) of thickness h. Intermediate layer of thickness H is a transversely isotropic poroelastic solid (medium II) and is resting on a cracked elastic solid half-space (medium III). A system of rectangular Cartesian coordinates is chosen with z-axis in the direction of increasing depth. Surface $z = 0$ represents the interface between liquid layer and poroelastic solid layer. Hence liquid layer occupies the region $-h < z < 0$. The transversely isotropic poroelastic solid occupies the region $0 < z < H$ and the region $z > H$ is occupied by cracked elastic solid half-space, as shown in Fig. 1.

3. Formulation of the problem

The objective is to study the dispersion of Stonely type surface waves during an earthquake preparation process. Such a process is represented by continuous accumulation of stress around focal region of eventual failure. Modifications of the cracks are the most direct effects of accumu-

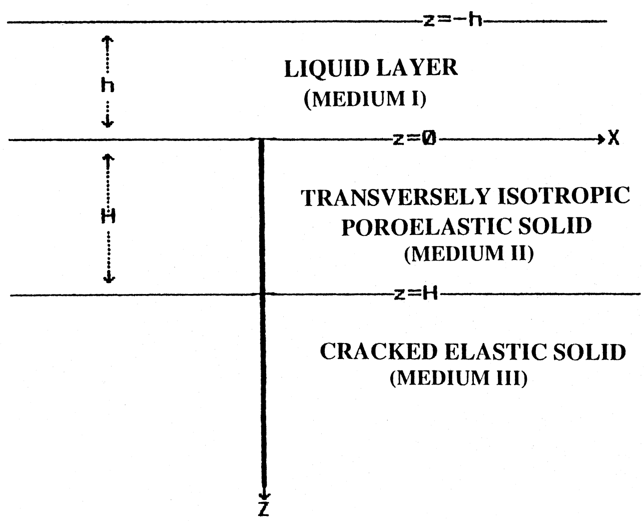

Fig. 1. Geometry of the medium.

lation of stress before an earthquake. Modifications may be the changes in orientation, density and thickness of the cracks. Precursors observed are caused by these changes in crack parameters. I propose to check the possible changes in dispersion behaviour, prior to an earthquake, as a precursor. Two-layered model (Fig. 1) of oceanic crust is considered with region of eventual failure in the lower-most half-space of cracked elastic solid. Sedimentary layer of ocean bottom is assumed a transversely isotropic water saturated porous solid. Anisotropy present in this medium may be due to the settlement of sediments in preferred orientations and presence of vertically aligned microcracks (Crampin, 1981). Lower-most half-space (basaltic rock) is assumed a transversely isotropic elastic solid. Anisotropy parameters for this medium are dependent upon the density and aspect ratio (ratio of thickness to diameter of a circular crack) of the cracks present. Crack modifications due to stress accumulation are represented by changes in crack density and aspect ratio.

4. Displacement and stress components

Consider the two-dimensional problem of wave propagation in x–z plane. Components along y-axis will vanish. Hence displacements are given by $(u_x, 0, u_z)$ and non-zero stress components, on the planes with normal in z-direction, will be σ_{xz} and σ_{zz}.

4.1. Medium I

For a liquid layer with $z = -h$ as its stress free surface, the displacement potential ϕ_0 for a compressional wave is given by

$$\phi_0 = A_0[\exp\{k\xi_0(z+h)\} + \exp\{-k\xi_0(z+h)\}\exp\{ik(x-ct)\}] \tag{1}$$

where k denotes horizontal wave number, c is phase velocity and $\xi_0 = \sqrt{1-(c^2/v_0^2)}$. Velocity of compressional wave $v_0 = \sqrt{K_0/\rho_0}$, where K_0 and ρ_0 denote bulk modulus and density of the liquid, respectively.

Displacement and stress components are expressed as

$$u_x = \partial\phi_0/\partial x, \quad u_z = \partial\phi_0/\partial z; \tag{2}$$

$$\sigma_{zx} = 0, \quad \sigma_{zz} = K_0[\partial^2\phi_0/\partial x^2 + \partial x^2\phi_0/\partial z^2]. \tag{3}$$

4.2. Medium II

Following Sharma and Gogna (1991), in a transversely isotropic liquid saturated porous solid, the displacements in solid part $(u_x, 0, u_z)$ and in liquid part $(U_x, 0, U_z)$ are given by

$$u_x = \sum_{n=1}^{6} f(n)a_1(n)E(n), \quad u_z = \sum_{n=1}^{6} f(n)a_3(n)E(n), \tag{4}$$

$$U_x = \sum_{n=1}^{6} f(n)b_1(n)E(n), \quad U_z = \sum_{n=1}^{6} f(n)b_3(n)E(n), \tag{5}$$

where $f(n)$, $(n = 1, 2, \ldots, 6)$, are relative excitation factors. $E(n)$ is expressed as

$$E(n) = \exp\{ik(x-ct)+kr(n)z\}, \tag{6}$$

where, $r(n)$, $(n = 1, 2, \ldots, 6)$ denote the slowness values (product of phase velocity, c, and vertical slowness) for three upgoing and three downgoing quasi waves. The constants a_1, a_3, b_1 and b_3, the functions of $r(n)$, are defined in the Appendix.

The stress components in solid and liquid parts are as follows,

$$\begin{aligned}\sigma_{zx} &= L(\partial u_z/\partial x + \partial u_x/\partial z),\\ \sigma_{zz} &= F\,\partial u_x/\partial x + C\,\partial u_z/\partial z + Q(\partial U_x/\partial x + \partial U_z/\partial z),\\ \sigma &= M\,\partial u_x/\partial x + Q\,\partial u_z/\partial z + R(\partial U_x/\partial x + \partial U_z/\partial z).\end{aligned} \tag{7}$$

A, C, F, L, M, N, Q, and R are elastic constants for transversely isotropic poroelastic solid (Biot, 1956, 1962). Following Thomsen (1986) anisotropic parameters $(\varepsilon, \gamma, \delta)$ are defined by

$$2\varepsilon = \frac{A+2N}{C} - 1; \quad 2\gamma = \frac{N}{L} - 1; \quad 2\delta = \frac{(F+L)^2-(C-L)^2}{C(C-L)}. \tag{8}$$

On similar lines an anisotropic parameter (say χ) is defined for liquid–solid coupling as $2\chi = M/Q - 1$. These relations enable one to represent the elastic properties of this medium by the numerical value of A, N, Q, R, ε, γ, δ and χ.

4.3. Medium III

Following Sharma et al. (1991), the displacements for the surface waves in a transversely isotropic elastic solid half-space are written as

$$u_x = \{P_1 \exp(-ks_1 z) + P_2 \exp(-ks_2 z)\} \exp\{ik(x-ct)\},$$
$$u_z = \{m_1 P_1 \exp(-ks_1 z) + m_2 P_2 \exp(-ks_2 z)\} \exp\{ik(x-ct)\}, \tag{9}$$

where P_1 and P_2 are arbitrary constants. s_1 and s_2 denote the slowness for quasi-P and quasi-SV waves, respectively. If C_{11}, C_{13}, C_{33}, C_{44}, C_{66} are elastic constants and ρ is density for this medium then

$$m_j = (C_{44} s_j^2 + R^*)/(is_j J^*), \quad (j = 1, 2), \tag{10}$$

$$s_j^2 = [-\Gamma + (-1)^j \sqrt{\Gamma^2 - 4C_{44}C_{33}R^*S^*}]/(2C_{44}C_{33}), \quad (j = 1, 2), \tag{11}$$

where

$$J^* = C_{44} + C_{13}, \quad R^* = \rho c^2 - C_{11}, \quad S^* = \rho c^2 - C_{44}, \tag{12}$$

and

$$\Gamma = R^* C_{33} + S^* C_{44} + (J^*)^2. \tag{13}$$

The stress components are given by

$$\sigma_{zx} = C_{44}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right), \quad \sigma_{zz} = C_{13}\frac{\partial u}{\partial x} + C_{33}\frac{\partial u_z}{\partial z}. \tag{14}$$

Anisotropy in the medium is caused by the presence of vertically aligned parallel cracks. Following Thomsen (1995), the anisotropy parameters $(\varepsilon', \gamma', \delta')$ are related to crack density (η) and crack porosity (ϕ_c) of the parallel cracks present in the elastic solid. These relations are defined as follows.

$$2\varepsilon' = \left(\frac{E}{\bar{E}} - 1\right) \Big/ (1 - v^2)\,; \quad 2\gamma' = \frac{\mu}{\bar{\mu}} - 1\,; \quad 2\delta' = \frac{\Delta}{(1-v)(1-\Delta)}, \tag{15}$$

with

$$\Delta = \frac{\mu}{\bar{\mu}}\left(\frac{E}{\bar{E}} - 1\right)\frac{1-v}{1+v} + \left(\frac{\mu}{\bar{\mu}} - 1\right)(1 - 2v), \tag{16}$$

where, E and μ denote the Young's modulus and rigidity modulus, respectively, for the elastic solid, in the absence of cracks. v is Poisson's ratio for the solid grains. Barred quantities are the corresponding parameters in the cracked elastic solid. Effect of cracks on the elastic constants, used in (15)–(16) are defined as

$$\frac{E}{\bar{E}} = 1 + \eta\frac{16}{3}\left(1 - \frac{K_f}{K_s}\right)(1 - v^2)D_c, \quad \frac{\mu}{\bar{\mu}} = 1 + \eta\frac{16}{3}\frac{1-v}{2-v}, \tag{17}$$

where, K_f, K_s denote bulk moduli of the fluid and solid, respectively and $D_c^{-1} = 1 - (K_f/K_s)(1 - \frac{16}{9}(\eta/\phi_c)[(1-v^2)/(1-2v)])$. In relation to aspect ratio (c/a) of circular cracks (of radius a and thickness c), crack porosity, $\phi_c = \frac{4}{3}\pi\eta(c/a)$. Anisotropic parameters are related to elastic constants as

$$2\varepsilon' = \frac{C_{11}}{C_{33}} - 1, \quad 2\gamma' = \frac{C_{66}}{C_{44}} - 1, \quad 2\delta' = \frac{(C_{13}+C_{44})^2}{C_{33}(C_{33}-C_{44})} + \frac{C_{44}}{C_{33}} - 1. \tag{18}$$

The relations (15)–(18) enable one to represent this medium with the numerical values of C_{33}, C_{44}, K_f, K_s, v, η and ϕ_c.

5. Boundary conditions

Following Deresiewicz and Skalak (1963), the boundary conditions appropriate for the interface between liquid and liquid saturated porous solid are the continuity of stress components, liquid pressure and normal component of displacement. For two-dimensional motion in x–z plane, at $z = 0$, these are

$$\begin{aligned}
(\sigma_{zz})_{II} + (\sigma)_{II} &= (\sigma_{zz})_{I}, \\
(\sigma_{zx})_{II} &= 0, \\
(\sigma)_{II} &= \phi_p(\sigma_{zz})_{I}, \\
(1-\phi_p)(u_z)_{II} + \phi_p(U_z)_{II} &= (u_z)_{I}.
\end{aligned} \tag{19}$$

ϕ_p is porosity of the poroelastic solid (Medium II).

At $z = H$, i.e., interface between liquid saturated porous solid and elastic solid, the appropriate boundary conditions, following Deresiewicz and Skalak (1963), are the continuity of stress and displacement components. A condition restricting the flow of liquid from poroelastic solid to elastic solid is also considered. These are given by

$$\begin{aligned}
(\sigma_{zz})_{III} &= (\sigma)_{II} + (\sigma_{zz})_{II}, \\
(\sigma_{zx})_{III} &= (\sigma_{zx})_{II}, \\
(u_x)_{III} &= (u_x)_{II}, \\
(u_z)_{III} &= (u_z)_{II}, \\
(u_z)_{II} &= (U_z)_{II}.
\end{aligned} \tag{20}$$

6. Dispersion equation

Making use of relations (2)–(5), (7), (9) and (14) in the boundary conditions defined by (19) and (20), a system of nine homogeneous equations in $f(n)$, $(n = 1, 2, \ldots, 6)$; P_1, P_2 and A_0 is obtained. Non-trivial solution of this system of equations requires a determinantal equation to be satisfied. The equation is

$$\mathrm{Det}\,\{a_{lm}\} = 0, \tag{21}$$

where a_{lm} are elements of a square matrix of order 9. These elements are defined as follows.

For $m = 1, 2, \ldots, 6$:

$$a_{1m} = \{(C+Q)a_3(m)+(Q+R)b_3(m)\}r(m)+(F+M)a_1(m)+(Q+R)b_1(m),$$
$$a_{2m} = a_1(m)r(m)-a_3(m),$$
$$a_{3m} = \{Qa_3(m)+Rb_3(m)\}r(m)+Ma_1(m)+Rb_1(m),$$
$$\alpha_{4m} = a_3(m)+\phi_{\rm p}\{b_3(m)-a_3(m)\}, \quad a_{5m} = a_{1m}X(m), \quad a_{6m} = a_{2m}X(m),$$
$$a_{7m} = a_{3m}X(m), \quad a_{8m} = a_1(m)X(m), \quad a_{9m} = \{a_3(m)-b_3(m)\}C(m), \tag{22}$$

where, $X(m) = \exp\{r(m)kH\}$.

For $m = 7, 8$:

$$a_{lm} = 0,(l = 1,2,3,4); \quad a_{57} = -R_1, \quad a_{58} = -R_2; \quad a_{67} = Q_1, \quad a_{68} = Q_2;$$
$$a_{77} = S_1, \quad a_{78} = S_2; \quad a_{8m} = -1; \quad a_{9m} = 0, \tag{23}$$

where

$$R_j = C_{13}+\frac{C_{33}}{J^*}(C_{44}s_j^2+R^*), \quad Q_j = C_{44}\frac{(C_{13}s_j^2-R^*)}{s_jJ^*L}, \quad S_j = \frac{(C_{44}s_j^2+R^*)}{s_jJ^*}. \tag{24}$$

Remaining elements are

$$a_{19} = K_0S_0'\frac{c^2}{v_0^2}, \quad a_{29} = 0, \quad a_{39} = \phi_{\rm p}a_{19}, \quad a_{49} = -S_0'', \quad a_{l9} = 0;(l = 5,6,\ldots,9), \tag{25}$$

where for $c < v_0: S_0' = \exp(kh\xi_0)-\exp(-kh\xi_0)$; $S_0'' = \xi_0\{\exp(kh\xi_0)+\exp(-kh\xi_0)\}$; and for $c \geqslant v_0$ $(\xi_0' = i\xi_0): S_0' = \sin(kh\xi_0')$; $S_0'' = \xi_0'\cos(kh\xi_0')$.

Equation (21) is the frequency equation for the propagation of Stoneley waves in the two-layered model of oceanic crust. This equation can be represented as $F(c,kH,kh) = 0$ and hence indicates the dispersive nature of existing surface waves. Such a surface wave can exist if for given values of dimensionless quantities kH and kh, a real value of phase velocity c is found satisfying this frequency equation and when

(i) s_j^2 $(j = 1,2)$, given by (11) are real and positive;
(ii) $r(m)$ $(m = 1,2,\ldots,6)$, used in (6), are either purely real or imaginary.

6.1. Special cases

(i) Uppermost layer (uniform liquid) can be eliminated by reducing the thickness h of this layer to zero. This modified model represents the continental crust and particularly is suitable for Himalayan type crust where the upper layer consists of marine sediments.
(ii) Middle layer (poroelastic solid) can be eliminated by reducing the thickness H of this layer to zero. The frequency eqn (21) then represents the dispersion of Stoneley waves in a cracked elastic solid lying under the uniform layer of liquid. Such a model represents the oceanic crust around mid-ocean ridges where the sediments are absent.
(iii) Substituting both h and H by zero, the problem reduces to the propagation of Rayleigh waves in a transversely isotropic elastic solid half-space with anisotropy depending upon crack parameters. The frequency equation (21) reduces to $Q_1R_2-Q_2R_1 = 0$.

7. Numerical results

To solve the frequency eqn (21) and to study the dispersion of surface waves, numerical work is restricted to a particular model. The two layered model of oceanic crust is considered to be water saturated sandstone resting over basaltic bed rocks. Numerical values of relevant constants are assumed as follows:

7.1. Medium I

Uniform layer of in-viscid water is represented by

$$K_0 = 2.14\times10^{10}\ \text{dyne/cm}^2, \quad \rho_0 = 1\ \text{gm/cm}^3.$$

7.2. Medium II

Sedimentary layer of transversely isotropic poroelastic solid is represented by water saturated sandstone. The elastic and dynamical constants for water saturated sandstone (Yew and Jogi, 1976) and anisotropic parameters for Taylor sandstone (Thomsen, 1985) are given by

$$A = 3.06\times10^{10}\ \text{dyne/cm}^2 \quad \rho_{11} = 1.9032\ \text{gm/cm}^3$$
$$Q = 0.13\times10^{10}\ \text{dyne/cm}^2 \quad \rho_{12} = 0$$
$$R = 0.637\times10^{10}\ \text{dyne/cm}^2 \quad \rho_{22} = 0.268\ \text{gm/cm}^3$$
$$N = 9.22\times10^{10}\ \text{dyne/cm}^2 \quad \phi_p = 0.268$$
$$\varepsilon = 0.11, \quad \gamma = 0.255, \quad \delta = -0.035.$$

For simplicity solid–liquid coupling is assumed isotropic, i.e., $\chi = 0$.

7.3. Medium III

Lower-most solid half-space is a cracked (transversely isotropic) elastic solid. The elastic constants are derived from density and vertical speeds α and β of P and S waves, respectively. For upper pillows of basalt: $\alpha = 5$ km/s, $\beta = 2.75$ km/s and $\rho = 2.7$ gm/cm^3 are assumed. Anisotropic parameters are derived from K_f/K_s (ratio of bulk moduli of liquid in cracks and solid grains), v (Poisson's ratio of solid), η (crack density) and ϕ_c (crack porosity). It is assumed that $K_f/K_s = 0.053$ (water saturated cracks) and $v = 0.28$. Values of η and ϕ_c are varied to check the effects of variations of crack parameters on dispersion.

For the above mentioned values of various constants the frequency eqn (21) is solved for the smallest (i.e., fundamental mode) values of non-dimensional velocity c/v_0 (v_0 is the velocity of sound in ocean water) and for the given values of kH. Thickness of water layer is fixed by assuming a numerical value for h/H. Group velocity (U) is obtained numerically using the formula

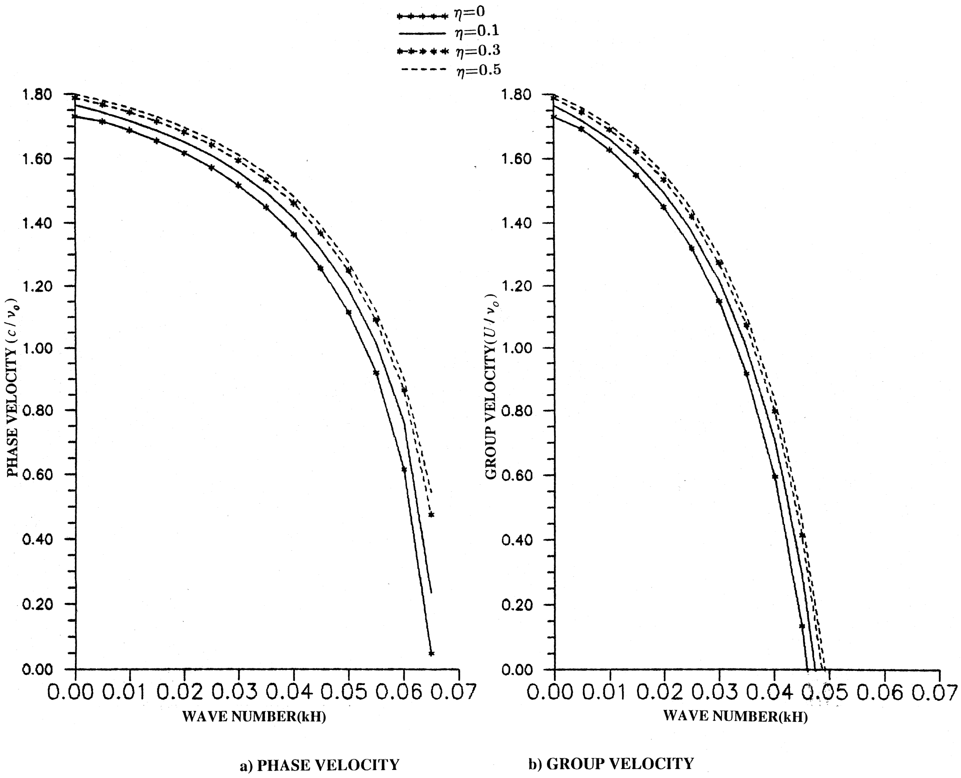

Fig. 2. Variations in dispersion with crack density. (a) Phase velocity. (b) Group velocity.

$$\frac{U}{v_0} = \frac{c}{v_0} + kH\frac{\mathrm{d}(c/v_0)}{\mathrm{d}(kH)}.$$

8. Discussion and conclusions

The layered model of crust is represented by the numerical values of various parameters as given in the previous section. In this particular type of oceanic crust Stoneley waves exist only for smaller values of dimensionless wave number kH (i.e., $kH < 0.07$). Figure 2 exhibits the variations in phase velocity and group velocity with kH for different values of crack density. Value of $c/a = 0.005$ and $h = 0.5$ (h/H and H denote the depth of ocean bottom and thickness of sediments, respectively). It is observed that increase in crack density increases the velocity of surface waves which is opposite to the behaviour of body waves. As expected, velocity of surface waves decreases with the increase of kH but this decrease is so fast that group velocity turns negative for $kH > 0.05$, i.e., dispersion is large enough to stop surface waves from carrying energy in the direction of propagation. Figure 3 shows the change in phase velocity and group velocity for different values of aspect ratio of cracks with $\eta = 0.2$ and $h/H = 0.5$. However these velocities increase with the increase of aspect ratio but this increase is negligible. Dispersion curves for different values of h/H (with $\eta = 0.2$ and

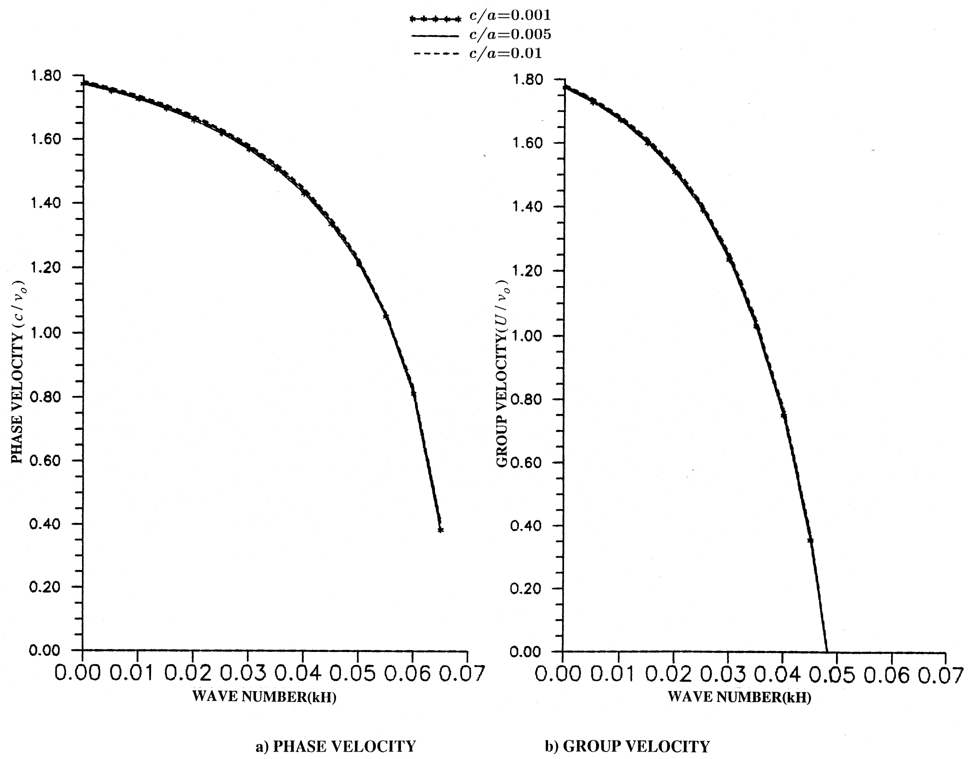

Fig. 3. Variations in dispersion with aspect ratio of cracks. (a) Phase velocity. (b) Group velocity.

$c/a = 0.005$) are shown in Fig. 4. It is observed that velocity of surface waves decreases slightly with the increase of h/H, only when either ocean is very deep or sediments are very thin. Velocity of Rayleigh waves in a transversely isotropic elastic solid (basalt) is found to be 1.78 times the velocity of sound in water.

It is concluded that

(i) Change in crack density has a significant effect on velocity of surface waves. For example, phase velocity may increase from 5% ($kH < 0.01$) to 50% ($kH > 0.05$) and group velocity may increase beyond 50% ($kH > 0.04$). Whereas, increase of crack density decreases the velocity of body waves, the velocity of surface waves increases with the increase in crack density. According to recent studies (Crampin and Zatsepin, 1997; Zatespin and Crampin, 1997) changes in crack density controls the fracture behaviour of rocks in the crust and hence significant dispersion changes can be expected during the stress accumulation in an earthquake preparation region.

(ii) The velocities of surface waves are not disturbed by the changes in aspect ratio of cracks. Studies indicate that the change in aspect ratio of cracks is the most likely change at the time of eventual failure in a focal region. Hence changes in dispersion should disappear as the failure process starts.

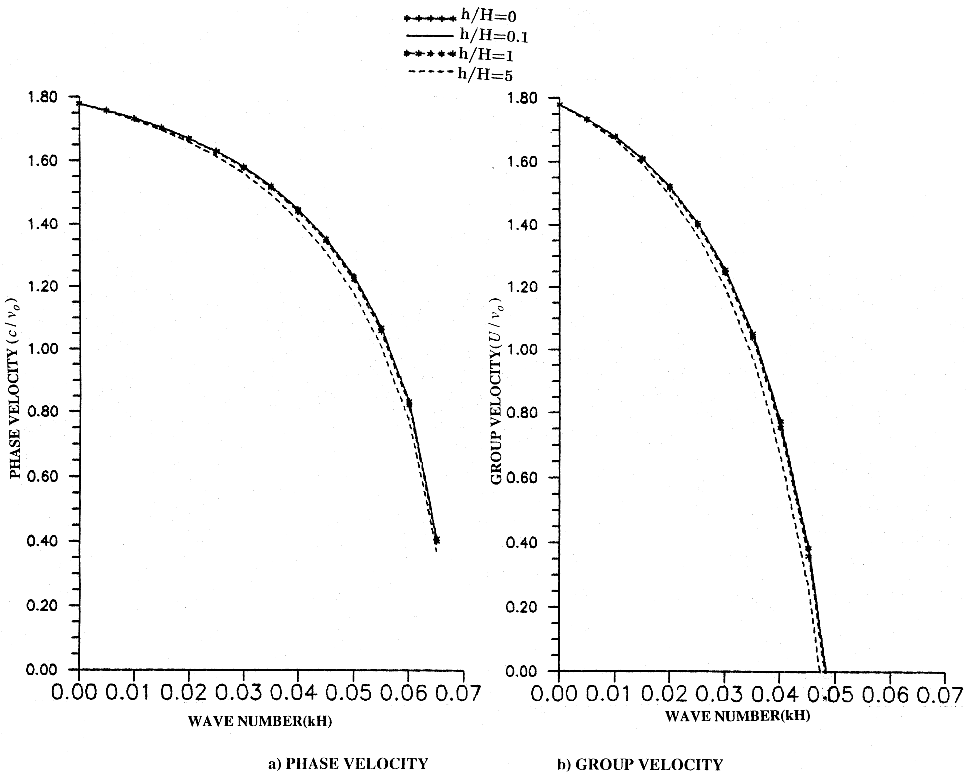

Fig. 4. Variations in dispersion with thickness of layers. (a) Phase velocity. (b) Group velocity.

(iii) Phase velocity and group velocity may not get any sudden change as surface waves propagate from continent to ocean or ocean to continent.
(iv) Only sudden and large increase (decrease) in ocean depth (sediment thickness) may affect the dispersion of surface waves.
(v) For a given wavenumber, existence of surface waves is more likely in thinner sediments.

The present study considers the surface wave propagation in a more realistic model of oceanic crust. Special cases are discussed to model the continental crust and crust around mid-ocean ridges. Dispersion variations with the changes in crack parameters may help in identifying an earthquake preparation region and to study crack modifications there.

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Appendix

Following Sharma and Gogna (1991), wave propagation in a transversely isotropic poroelastic solid is controlled by the cubic equation in a variable V (say). If V_1, V_2 and V_3 denote the real and positive roots of this equation then slowness values $r(m)$ $(m = 1, 2, \ldots, 6)$, are defined as

$$r(1) = \sqrt{V_1}, \quad r(2) = \sqrt{V_2}, \quad r(3) = \sqrt{V_3};$$
$$r(4) = -r(1), \quad r(5) = -r(2), \quad r(6) = -r(3).$$

This cubic equation is

$$T_0 V^3 - T_1 V^2 + T_3 V - T_4 = 0.$$

The coefficients T_j $(j = 0, 1, 2, 3)$, are defined as follows.

$$T_0 = \rho_{22} LX, \quad T_1 = T_{11} c^2 + T_{12},$$
$$T_2 = T_{21} c^4 + T_{22} c^2 + T_{23}, \quad T_3 = T_{31} c^6 + T_{32} c^4 + T_{33} c^2 + T_{34},$$

where

$$T_{11} = -(XZ + \rho_{22} LY),$$
$$T_{12} = \rho_{22}\{(2N+L)X + M[Q(F+2L) - CM] + Q(FM - AQ) + R[AC - F(F+2L)]\}$$
$$T_{21} = (Y + \rho_{22} L)Z,$$
$$T_{22} = \rho_{11}\rho_{22}[Q^2 + M^2 - (C+A+2N)R] - 2\rho_{12}^2[MQ - (F+2L)R] + 2\rho_{22}^2 (L-N)C + 2\rho_{12}\rho_{22}[(C+A+2N)M - 2(F+2L)Q] + \rho_{22}^2[F(F+2L) - AC]$$
$$T_{23} = \rho_{22}\{3NX + [AC - F(F+2L) + L(A+2N) - CN]R + (FM - AQ)Q + NQ^2 - LM^2 + (F+2L)MQ - CM^2\}$$
$$T_{31} = -Z^2, \quad T_{32} = -(Y' + \rho_{22} L)Z, \quad T_{33} = -(X'Z + \rho_{22} LY'), \quad T_{34} = \rho_{22} LX',$$

with

$$X = CR - Q^2, \quad Y = \rho_{11} R + \rho_{22} C - 2\rho_{12} Q, Z = \rho_{11}\rho_{22} - \rho_{12}^2,$$
$$X' = (A+2N)R - M^2, \quad Y' = \rho_{11} R + \rho_{22}(A+2N) - 2\rho_{12} M,$$

A, C, F, L, M, N, Q, R are elastic constants for transversely isotropic poroelastic solids. ρ_{11}, ρ_{12} and ρ_{22}, the dynamical parameters for poroelastic solids, are defined as

$$\rho_{11} + \rho_{12} = (1 - \phi_p)\rho_s, \quad \rho_{12} + \rho_{22} = \phi_p \rho_f;$$

where ρ_s, ρ_f denote the densities of solid grains and fluid, respectively and ϕ_p is porosity.

The coefficients $a_1(n)$, $a_3(n)$, $b_1(n)$, $b_3(n)$ used in eqns (4) and (5) are given by

$$a_1(n) = \frac{X_1(n)}{D(n)}, \quad a_3(n) = \frac{X_2(n)}{D(n)}, \quad b_1(n) = \frac{X_3(n)}{D(n)}, \quad b_3(n) = \frac{X_4(n)}{D(n)},$$

where,

$$\begin{aligned}
X_1(n) &= \{[r(n)]^4 X - [r(n)]^2[(LR+X) - c^2 Y] + Zc^4 + LR\}\rho_{22} - (ZR + \rho_{22}^2 L)c^2\,; \\
X_2(n) &= -[r(n)]^3[(F+L)R - MQ]\rho_{22} + r(n)\{(F+L)R - MQ \\
&\quad + [\rho_{12}\rho_{22}(M+Q) - \rho_{12}^2 R - \rho_{22}^2(F+L)]c^2\}\,; \\
X_3(n) &= -\rho_{12}\{[r(n)]^4 X - [r(n)]^2\{\rho_{22}[(F+L)Q - MC] + \rho_{12}[MQ \\
&\quad - (F+2L)R] + c^2\rho_{12} Y\} - \rho_{12} Zc^4 + (ZM + \rho_{12}\rho_{22}L)c^2 - \rho_{22} ML\,; \\
X_4(n) &= -[r(n)]^3\{[CM - (F+L)Q]\rho_{22} - \rho_{12} X\} + r(n)\{\rho_{12}[MQ \\
&\quad - (F+2L)R] + \rho_{22} LM + \rho_{12}[\rho_{11} R + \rho_{22}(F+L) - \rho_{12} Q]c^2 - \rho_{12}\rho_{22} Mc^2\}\,; \\
D(n) &= \sqrt{X_1^2(n) + X_2^2(n) + X_3^2(n) + X_4^2(n)}
\end{aligned}$$

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